DISSERTATIO ASTRONOMICA
DE
INVENIENDO MOMENTO CULMINATIONIS SOLIS VEL STELLÆ CUJUSDAM EX OBSERVATIS DUABUS VEL PLURIBUS IPSIUS ALTITUDINIBUS.

Quam

Conſ. Ampliſſ. Facult. Philoſ. Aboënſ.

PRÆSIDE

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ABOÆ, typis Frenckellianis.

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§. 1.

Corpora Cœlestia eo momento culminari dicuntur, quo meridiani partem superiorem transeunt. Ad Momentum hujus transitus, seu culminationis, ut etiam nuncupatur, inveniendum, varias adhibuerunt Astronomi methodos; illud vero præcipue vel ope ascensionum rectarum, vel ex observatis altitudinibus correspondentibus investigarunt. Quod autem ad has nominatas attinet methodos, quamvis earum ope ad exactitudinem desideratam pervenire possimus, variis tamen in praxi obnoxiæ sunt incommodis. Methodus enim altitudinum correspondentium, quamvis maxime directa esse videatur, eo tamen laborat incommodo, quod observationes ad certum restrictæ sint tempus, quo fit, ut nubilum Cœlum, variæque aëris vicissitudines, laborem haud raro irritum reddant. Neque momentum culminationis exacte satis habetur, nisi variationes declinationis Sideris observati in calculum revocatæ fuerint, quibus efficitur, ut aliquanto prolixior reddatur hæc via ad tempus

 ipsius

ipsius transitus inveniendum. Pari quoque correctione eget methodus illa, qua momentum culminationis Stellæ cujusdam ex ascensionibus rectis determinatur. Has enim ascensiones invariabiles assumere non licet, adeoque ipse etiam calculus diffusior evadit. His vero perspectis difficultatibus, aliam pro inveniendo momento culminationis rationem in *Samlung Astronomischer Abhandlungen, Beobachtungen und Nachrichten, herausgegeben von* J. E. BODE, *I:er Suppl. B. p. 214* dedit Nob. de TEMPELHOF, qua scilicet ex observatis duabus vel pluribus sideris cujusdam altitudinibus, datis, pro tempore observationis, declinationibus & Latitudine Loci, nec non intervallo temporis inter observationes præterlapso, momentum transitus, a horologio indicatum, determinatur. Hanc methodum eo ex capite commodam inprimis censemus, quod sumtis altitudinibus quibusdam ante & post meridiem, momentum culminationis toties determinari possit, quoties binæ ex istis ante & post meridiem factis observationibus combinari possunt, unde denique medium sumendo Arithmeticum, momentum quæsitum exacte satis habetur. Hoc igitur Problema exponere nobis proposuimus, L. B. censuræ quæ huc pertinent jam submittentes. Nimiam vero ut evitemus prolixitatem, observatas altitudines, mox debite correctas, & motum horologii in spatio 24^{h} æquabilem supponimus.

§. 2.

§. 2.

Ad Problema vero nostrum solvendum sequens nobis commodissima videtur methodus. Sumto videlicet in arcu ZP, P polo & Z Zenith, erit ZP complementum latitudinis loci. Observato Astro ante meridiem in A, & post meridiem in B, ductisque arcubus circulorum maximorum ZA, ZB, AP & BP, erunt ZA & ZB complementa altitudinum observatarum, BP autem & AP complementa declinationum sideris observati pro temporibus observationum, angulus vero APB horarius, intervallum temporis inter utramque observationem exhibebit. Sit deinde $AZ = 90° - A$, $BZ = 90° - \alpha$, $AP = 90° - D$, $BP = 90 - \Delta$, $ZP = 90° - L$, & bisecto angulo $APB = 2m$ arcu PM, si $> ZPM = \varphi$, erit $> APZ = m \mp \varphi$ & $> BPZ = m \pm \varphi$, prout scilicet ad unam vel alteram ipsius ZP partem cadat arcus PM. In Triangulo vero APZ habebitur, posito Sinu Toto $= 1$ (El. Trig. Sphær.) $Cos\, APZ = \dfrac{Cos\, AZ - Cos\, AP\; Cos\, ZP}{Sin\, AP\; Sin\, ZP}$, pariterque in Triangulo BPZ, $Cos\, BPZ =$

$$\frac{Cos\, BZ - Cos\, BP\, Cos\, ZP}{Sin\, BP\; Sin\, ZP} \text{ seu } Cos\,(m \mp \varphi) = \frac{Sin\, A - Sin\, D\; Sin\, L}{Cos\, D\; Cos\, L}$$

$$\& \; Cos\,(m \pm \varphi) = \frac{Sin\, \alpha - Sin\, \Delta\; Sin\, L}{Cos\, \Delta\; Cos\, L}.$$ Hinc vero $Cos\,(m \mp \varphi)$

 $- Cos$

$$= Cos\,(m \pm \varphi) = \frac{Sin\,A - Sin\,D\;Sin\,L}{Cos\,D\;Cos\,L} \mp \left(\frac{Sin\,\alpha - Sin\,\Delta\;Sin\,L}{Cos\,\Delta\;Cos\,L}\right) =$$

$$\frac{Sin\,A\,Cos\,\Delta - Sin\,D\,Sin\,L\,Cos\,\Delta - Sin\,\alpha\,Cos\,D + Sin\,\Delta\,Sin\,L\,Cos\,D}{Cos\,D\;\;Cos\,\Delta\;\;Cos\,L}$$

Quum vero generatim sit $Cos\,p - Cos\,q = 2\,Sin\,(\tfrac{1}{2}p + \tfrac{1}{2}q)\;Sin\,(\tfrac{1}{2}q - \tfrac{1}{2}p)$ & $Sin\,p\;Cos\,q - Cos\,p\;Sin\,q = Sin\,p - q$; habebitur facta reductione $2\,Sin\,m\;Sin \mp \varphi =$
$\frac{Sin\,A\,Cos\,\Delta - Sin\,\alpha\,Cos\,D + Sin\,L\,Sin\,(\Delta - D)}{Cos.\,D\;\;Cos\,\Delta\;\;Cos\,L}$; unde $Sin \mp \varphi$
$= \frac{Sin\,A\,Cos\,\Delta - Sin\,\alpha\,Cos\,D + Sin\,L\,Sin\,(\Delta - D)}{2\,Cos.\,D\;\;Cos\,\Delta\;\;Cos\,L\;\;Sin\,m}$.

Quo autem valor $Sin \mp \varphi$ Logarithmorum ope investigari queat, ponatur $\frac{Sin\,\alpha\;Cos\,D}{Sin\,A\;Cos\,\Delta} = Cos\,\psi^2$, & $\frac{Sin\,\psi^2\;Sin\,A\;Cos\,\Delta}{Sin\,L\;Sin\,(\Delta - D)}$ $= Tang.\,\xi^2$; erit facta substitutione $Sin \mp \varphi =$
$\frac{Tang.\,L\;Sin\,(\Delta - D)}{2\,Cos\,D\;Cos\,\Delta\;Cos\,\xi^2\;Sin\,m}$, ob $1 - Cos\,\psi^2 = Sin\,\psi^2$ &

$1 + Tang\,\xi^2 = \frac{1}{Cos.\,\xi^2}$.

Invento jam angulo φ, dabitur quoque angulus $m \mp \varphi$, qui in tempus convertatur inferendo $360^\circ : m \mp \varphi : : 24^h$ ad tempus quæsitum, quod, si addatur tempori quo Stella in A observata sit, exhibebit momentum culminationis. Idem

vero

vero tempus, si auferatur a hora ista, qua Sidus in B est observatum, dabit etiam momentum transitus.

COROLL. Quod si altitudines observatæ æquales fuerint, formula nostra pro angulo φ hanc induit formam: $Sin \mp \varphi$

$$= \frac{Sin\,A\,(Sin\,\overline{\tfrac{1}{2}D + \tfrac{1}{2}\triangle}\;Sin\,\overline{\tfrac{1}{2}D - \tfrac{1}{2}\triangle}) + Sin\,L\,Sin\,\overline{\triangle - D}}{Cos\,\triangle\;Cos\,D\;Cos\,L\;Sin\,m}.$$

Quo autem hæc æquatio faciliorem admittat logarithmorum usum, statuatur $\frac{Sin\,A\,(Sin\,\overline{\tfrac{1}{2}D + \tfrac{1}{2}\triangle}\;Sin\,\overline{\tfrac{1}{2}D - \tfrac{1}{2}\triangle})}{Sin\,L\,Sin\,(\triangle - D)}$ $= Tang.\,\xi^2$, eritque facta substitutione $Sin \mp \varphi =$ $\frac{Tang\,L\;Sin\,\overline{\triangle - D}}{Cos\,D\;Cos\,\triangle\;Cos\,\xi^2\;Sin\,m}$.

Existentibus autem declinationibus Stellæ observatæ æqualibus, habebitur $Sin \mp \varphi = \frac{Cos\,(\overline{\tfrac{1}{2}A + \tfrac{1}{2}\alpha}\;Sin\,\overline{\tfrac{1}{2}A - \tfrac{1}{2}\alpha})}{Cos\,D\;Cos\,L\;Sin\,m}$.

EXEMPL. 1. In Latitudine = 60°, 27′ 10″ die 5 Martii anni currentis, si fuerint observatæ hora ante meridiem 10 & post meridiem 3 altitudines 19° 15′ 55″ & 14° 40′ 21″ Solis, cujus pro utraque observatione declinationes 6° 14′ 34″ & 6° 9′ 45″ respective quoque dantur, momentum culminationis pro hoc eodem die a horologio indicatum his datis ita computabitur:

$A = 19°$

$A = 19^\circ\ 15'\ 55''$

$\alpha = 14^\circ\ 40'\ 21''$

$D = 6^\circ\ 14'\ 34''$

$\Delta = 6^\circ,\ 9',\ 45''$

$\Delta\text{-}D = -0^\circ\ 4'\ 49''$

$L = 60^\circ\ 27'\ 10''$

$2m = 75^\circ\ 0'\ 0''$

$m = 37^\circ 30'\ 0''$

$-\varphi = 7^\circ\ 30'\ 1'',4$

$m\text{-}\varphi = 29^\circ 59^\circ\ 58''6$

$Sin\ \alpha = \bar{1},4036283$

$Cof\ D = \bar{1},9974170$

$-Cof\ \Delta = 0,0025168$

$-Sin\ A = 0,4815634$

$Cof\ \psi^2 = \bar{1},8851255$

$Sin\ \psi^2 = \bar{1},3662626$

$Sin\ A = \bar{1},5184366$

$Cof\ \Delta = \bar{1},9974832$

$-Sin\ L = 0,0605059$

$-Sin\Delta\text{-}D = 2,8535274$

$Tang\ \xi^2 = \bar{1},7962157$

$Tang\ L = 0,2465232$

$Sin\Delta\text{-}D = \bar{3},1464726$

$-Cof\ \Delta = 0,0025168$

$-Cof\ D = 0,0025830$

$-Sin\ m = 0,2155529$

$-Log\ 2 = \bar{1},6989700$

$-Cof\ \xi^2 = \bar{1},8031012$

$Sin \mp \varphi = \bar{1},1157197$

Inferendo deinde $360^\circ : 29^\circ\ 59'\ 58''$, $6 : : 24^h :$ $1^h\ 59'\ 59'',9067$, habebitur tempus, quod, si tempori primæ observationis 10^h addatur exhibet momentum transitus $11^h\ 59'\ 59'',9067$.

EXEMPL. 2. Eodem loco & die hora ante meridiem 11 & post meridiem 2, observatæ sunt altitudines Solis

lis 22° 16′ 57″, & 19° 19′ 37″, datis insimul declinationibus 6° 13′ 36″ & 6° 10′ 43″ utrique observationi respective respondentibus: momentum culminationis modo sequenti investigatur:

$A = 22° \; 16' \; 57''$	$Sin\ \alpha = \bar{1},5197774$
$\alpha = 19° \; 19' \; 37''$	$Cos\ D = \bar{1},9974303$
	$-Cos\ \Delta = 0,0025301$
$D = 6° \; 13' \; 36''$	$-Sin\ A = 0,4211640$
$\Delta = 6° \; 10' \; 43''$	$Cos\ \psi^2 = \bar{1},9409018$
$\Delta\text{-}D = -0° \; 2' \; 53''$	$Sin\ \psi^2 = \bar{1},1045774$
$L = 60°.\ 27'.\ 10$	$Sin\ A = \bar{1},5788360$
	$Cos\ \Delta = \bar{1},9974699$
$2m = 45° \; 0' \; 0''$	$-Sin\ L = 0,0605059$
$m = 22° \; 30' \; 0''$	$-Sin\ \overline{\Delta\text{-}D} = 3,0763791$
$-\varphi = 7° \; 29' \; 59'',8$	$Tang\ \xi^2 = \bar{1},8177683$
	$Tang\ L = 0,2465232$
$m-\varphi = 15°,\ 0',\ 0'',2.$	$Sin\ \overline{\Delta\text{-}D} = \bar{4},9236209$
	$-Cos\ \Delta = 0,0025301$
	$-Cos\ D = 0,0025697$
	$-Sin\ m = 0,4171603$
	$-Log\ 2 = \bar{1},6989700$
	$-Cos\ \xi^2 = \bar{1},8343102$
	$Sin \mp \varphi = \bar{1},1156844$

Quum autem sit $360° : 15°, 0', 0,2'' : : 24^h : 1^h, 0', 0'', 0001$ habebitur momentum culminationis a horologio indicatum

tum (addendo scilicet $1^h . 0'. 0, 0001''$ ad 11^h) $12^h . 0'. 0,0001''$. Si vero combinentur observationes ad 10^h & 2^h factæ, habebitur momentum transitus $11^h, 59', 59'',_{94}$ & combinatæ observationes hora ante meridiem 11 & post meridiem 3, exhibent momentum culminationis $11^h, 59', 59'',_{7933}$. Medium autem omnium momentorum inventorum sumendo, prodit $11^h, 59', 59'',_{91005}$.

§. 3.

Quamvis methodus inveniendi momentum culminationis, quam explicuimus, eo imprimis sese commendet, quod errores in altitudinibus observatis Sideris cujusdam, minimum in momentum quæsitum habeant effectum; (medium enim sumendo arithmeticum omnium momentorum determinatorum, efficitur, ut ipsi errores se invicem destruant) a re tamen non est alienum examinare, qvantus ex dato errore in altitudinibus observatis, proveniat error in momento culminationis determinando. Positis itaque φ, A & α variabilibus, manentibus reliquis invariatis, resumatur æquatio $Sin \mp \varphi = \frac{Sin\, A\, Cos\, \triangle - Sin\, \alpha\, Cos\, D + Sin\, L\, Sin\, \overline{\triangle - D}}{2\, Cos\, D\, Cos\, \triangle\, Cos\, L\, Sin\, m}$, quæ si differentietur, obtinetur $Cos\, \varphi\, d\varphi = \frac{Cos\, \triangle\, Cos\, A\, d\, A\, -}{2\, Cos\, D\, Cos\, \triangle}$ $\frac{-\, Cos\, D\, Cos\, \alpha\, d\, \alpha}{Sin\, m\, Cos\, L}$, adeoque $d\varphi = \frac{Cos \triangle Cos A d A - Cos D Cos \alpha d \alpha}{2 Cos D Cos \triangle Sin\, m\, Cos\, L Cos\, \varphi}$.

Potest

Potest autem ratio inter $d\varphi$, dA & $d\alpha$ etiam sequenti modo investigari. Sumto videlicet in arcu meridiani ZP ($=90^\circ - L$) (Fig. 2) P polo & Z Zenith, sint loca Sideris observatæ A & B, vera autem A' & B', ductisque arcubus circulorum maximorum $AP = A'P = 90^\circ - D$, $BP = B'P = 90^\circ - \Delta$, $AZ = 90^\circ - A$, $A'Z = AZ \pm A'R = 90^\circ - A \pm dA$, $BZ = 90^\circ - \alpha$, $B'Z = BZ \pm B'N = 90^\circ - \alpha \pm d\alpha$, descriptis scilicet polis P & Z arcubus $A'A$ & $B'B$, AR & BN respective; bisecentur $>BPA$ & $>B'PA'$ arcubus PM & PM', eritque $>A'AR = >PAZ$, $>B'BN = >PBZ$, existentibus $>PAA' = >RAZ = 90^\circ$ & $>PBB' = >NBZ = 90^\circ$, $>APA' = >M'PM = BPB' = d\varphi$, ob invariatum angulum APB. Est vero in Triangulo $A'AR$, $AA' : A'R\ (= \pm dA) :: 1 : \mathrm{Sin}\, A'AR = \mathrm{Sin}\, PAZ$ (substitutis loco Sinuum ipsis arcubus, utpote Sinubus, in Triangulo admodum exiguo $A'AR$, æqualibus). In Triangulo $A'PA$ erit $\mathrm{Cos}\, D : A'A :: 1 : APA'\ (= d\varphi)$; unde componendo eruitur $\mathrm{Cos}\, D : \pm dA :: 1 : \pm d\varphi\, \mathrm{Sin}\, PAZ$. Hinc itaque $\pm dA = \mathrm{Cos}\, D\, \mathrm{Sin}\, PAZ\, d\varphi$. Pariter ex Triangulis $B'BN$ & $B'PB$ deducitur analogia $\mathrm{Cos}\, \Delta : \pm d\alpha :: 1 : d\varphi\, \mathrm{Sin}\, PBZ$, unde $\pm d\alpha = \mathrm{Cos}\, \Delta\, \mathrm{Sin}\, PBZ\, d\varphi$ & $\pm dA \pm d\alpha = d\varphi(\mathrm{Cos}\, \Delta\, \mathrm{Sin}\, PBZ + \mathrm{Cos}\, D\, \mathrm{Sin}\, PAZ)$ adeoque $d\varphi = \dfrac{\pm dA \pm d\alpha}{\mathrm{Cos}\, D\, \mathrm{Sin}\, PAZ + \mathrm{Cos}\, \Delta\, \mathrm{Sin}\, PBZ}$. Quum vero sit $\mathrm{Cos}\, D : \mathrm{Cos}\, L :: \mathrm{Sin}\, AZP : \mathrm{Sin}\, PAZ$, & $\mathrm{Cos}\, \Delta : \mathrm{Cos}\, L :: \mathrm{Sin}\, BZP : \mathrm{Sin}\, PBZ$, erit $\mathrm{Cos}\, D\, \mathrm{Sin}\, PAZ = \mathrm{Cos}\, L\, \mathrm{Sin}\, AZP$, & $\mathrm{Cos}\, \Delta\, \mathrm{Sin}\, PBZ = \mathrm{Cos}\, L\, \mathrm{Sin}\, PZB$, habebiturque $d\varphi =$

$$\frac{\pm dA \pm d\alpha}{Cos\,L\,(Sin\,AZP + Sin\,BZP)} = \frac{\pm dA \pm d\alpha}{2\,Cos\,L\,(Cos\,\tfrac{1}{2}\,AZP - \tfrac{1}{2}\,BZP)\,Sin\,(\tfrac{1}{2}\,AZP + \tfrac{1}{2}\,BZP)}.$$

§. 4.

Quod si vero errores quidam declinationes sideris observati, afficiant, methodo plane simili effectum ipsorum in momentum culminationis determinari, potest. Æquationem namque $Sin \mp \varphi =$

$$\frac{Sin\,A}{2\,Cos\,D\,Cos\,L\,Sin\,m} - \frac{Sin\,\alpha}{2\,Cos\,\Delta\,Cos\,L\,Sin\,m} + \frac{Tang\,L\,Sin\,\overline{\Delta - D}}{2\,Cos\,\Delta\,Cos\,D\,Sin\,m},$$

positis solummodo quantitatibus, D, Δ & φ variabilibus, reliquis vero constantibus, differentiando, habebitur

$$Cos\,\varphi\,d\varphi = \frac{d\Delta\,Sin\,\Delta\,Sin\,\alpha}{2\,Cos\,\Delta^2\,Cos\,L\,Sin\,m} - \frac{dD\,Sin\,D\,Sin\,A}{2\,Cos\,D^2\,Cos\,L\,Sin\,m} + Tang\,L$$

$$\left(\frac{\overline{d\Delta - dD}.\,Cos\,\overline{\Delta - D}.\,Cos\,\Delta\,Cos\,D + Sin\,\overline{\Delta - D}.\,Cos\,D\,Sin\,\Delta\,d\Delta +}{2\,Cos\,\Delta^2\,Cos\,D^2}\right.$$

$$\left.\frac{Cos\,\Delta\,Sin\,D\,dD}{Sin\,m}\right) = \frac{d\Delta\,Tang\,\Delta\,Sin\,\alpha}{2\,Cos\,\Delta\,Cos\,L\,Sin\,m} - \frac{dD\,Tang\,D\,Sin\,A}{2\,Cos\,D\,Cos\,L\,Sin\,m}$$

$$+ Sin\,L \left(\frac{\overline{d\Delta - dD}.\,Cos\,\overline{\Delta - D} + Sin\,\overline{\Delta - D}.\,\overline{Tang\,\Delta\,d\Delta + Tang\,D\,dD}}{2\,Cos\,\Delta\,Cos\,D\,Cos\,L\,Sin\,m}\right),$$

unde

$$d\varphi = \frac{d\Delta\,Tang\,\Delta\,Cos\,D\,Sin\,\alpha - dD\,Tang\,D\,Cos\,\Delta\,Sin\,A +}{2\,Cos\,\Delta\,Cos\,D\,Cos\,\varphi}$$

$$\frac{Sin\,L\,(\overline{d\Delta - dD}.\,Cos\,\overline{\Delta - D} + Sin\,\overline{\Delta - D}.\,\overline{Tang\,\Delta\,d\Delta + Tang\,D\,dD})}{Cos\,L\,Sin\,m}$$

Faci-

Facilius autem ratio ista detegitur ponendo (Fig. 1) A' & B' loca Stellæ vera, ductisque arcubus circulorum maximorum $A'Z$ & $B'Z$, $A'P$ & $B'P$ & descriptis Polis P & Z arcubus AR & BN, $A'A$ & $B'B$. Erit enim ob $\angle A'AZ = \angle PAR = 90°$, $RAA' = PAZ$, pariterque $\angle NBB' = \angle PBZ$, $\angle RPA = \angle NPB = M'PM$ existente $\angle APB$ invariato; $A'P = AP \pm dD = 90° - D \pm dD$, $B'P = BP \pm d\Delta = 90° - \Delta \pm d\Delta$, $A'Z = AZ$ & $B'Z = BZ$. In Triangulo $A'RA$ ad R rectangulo est $1 : AR :: Tang\, RAA'$ ($= Tang\, PAZ$) $: RA' = \pm dD$ & in Triangulo RPA, $RA : PAZ$ ($= d\varphi$) $::$ Sin AP ($= Cos\, D$) $: 1$, unde $1 : d\varphi :: Cos\, D\, Tang\, PAZ : \pm dD = d\varphi\, Cos\, D\, Tang\, PAZ$. Eodem modo ex Triangulis $B'BN$ & BPN eruitur analogia $1 : d\varphi :: Cos\, \Delta\, Tang\, PBZ : \pm d\Delta = d\varphi\, Cos\, D\, Tang\, PBZ$ adeoque $\pm dD \pm d\Delta = d\varphi\, (Cos\, D\, Tang\, PAZ \mp Cos\, \Delta\, Tang\, BPZ)$ & $d\varphi = \dfrac{\pm dD \pm d\Delta}{Cos\, D\, Tang\, PAZ \mp Cos\, \Delta\, Tang\, PBZ}$.

§. 5.

Restat vero quantitatem $d\varphi$ in casu, quo Latitudo Loci data, erronea fuerit, determinare. Resumta itaque æquatione $Sin \mp \varphi = \dfrac{Sin\, A\, Cos\, \Delta - Sin\, \alpha\, Cos\, D}{2\, Cos\, D\, Cos\, \Delta\, Cos\, L\, Sin\, m} \mp \dfrac{Tang\, L\, Sin\, \overline{\Delta - D}}{2\, Cos\, D\, Cos\, \Delta\, Sin\, m}$, ponantur φ & L variabiles, & habebi-

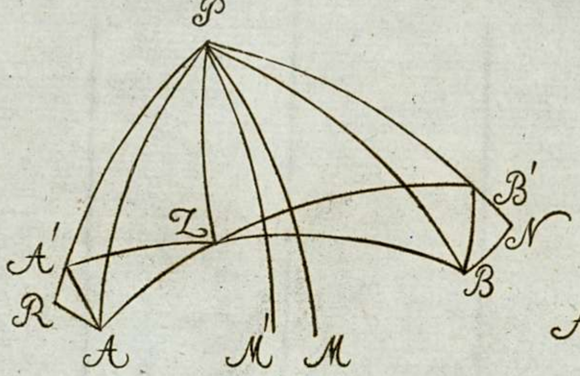

Fig. I.

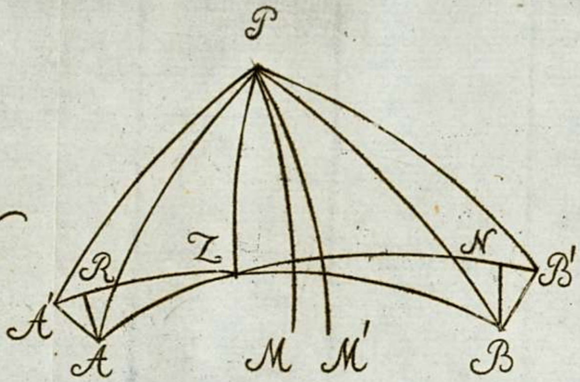

Fig. 2.

Gezelius Sc:

bebitur sumtis differentialibus $Cos\ \varphi\, d\varphi = -\frac{dL\ Sin\ L}{Cos\ L^2}$

$$\left(\frac{Sin\ A\ Cos\ \triangle - Sin\ \alpha\ Cos\ D}{2\ Cos\ D\ Cos\ \triangle\ Sin\ m}\right) + \frac{dL}{Cos\ L^2} \cdot \frac{Sin\ \overline{\triangle - D}}{2\ Cos\ D\ Cos\ \triangle\ Sin\ m},$$

adeoque $d\varphi = \frac{-dL}{Cos\ L^2}\left(\frac{Sin\ L.\overline{Sin\ A\ Cos\ \triangle - Sin\ \alpha\ Cos\ D} + Sin\ \overline{\triangle - D}}{2\ Cos\ D\ Cos\ \triangle.\ Cos\ \varphi\ Sin\ m}\right)$,

qua æquatione dato errore Latitudinis facillime innotescit valor ipsius $d\varphi$.